

# A tight scaling relation of dark matter in galaxy clusters

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## ABSTRACT

Recent studies in different types of galaxies reveal that the product of the central density and the core radius ( $\rho_c r_c$ ) is a constant. However, some empirical studies involving galaxy clusters suggest that the product  $\rho_c r_c$  depends weakly on the total dark halo mass. In this article, we re-analyse the hot gas data from 106 clusters and obtain a surprisingly tight scaling relation:  $\rho_c \propto r_c^{-1.46 \pm 0.16}$ . This result generally agrees with the claims that  $\rho_c r_c$  is not a constant for all scales of structure. Moreover, this relation does not support the velocity-dependent cross section of dark matter if the core formation is due to the self-interaction of dark matter.

**Key words:** Dark matter

## 1 INTRODUCTION

The dark matter problem is one of the key issues in modern astrophysics. The existence of cold dark matter (CDM) particles is the generally accepted model to tackle the dark matter problem. N-body simulations show that the dark matter density should follow a universal density profile (NFW profile), which goes like  $r^{-\alpha}$  towards the center of the structure with  $\alpha \sim 1 - 1.5$  (Navarro et al. 1997; Moore et al. 1999). However, observations in many dwarf galaxies and a few clusters indicate that flat cores of dark matter exist in those structures (Tyson et al. 1998; Gentile et al. 2004; Sand et al. 2008; deBlok 2010; Newman et al. 2011). This discrepancy can be reconciled in many possible scenarios. For example, the feedback from baryonic processes such as supernova explosion can generate core-like structure in dark matter profile (Weinberg and Katz 2002; Macciò et al. 2012). However, some studies point out that these processes cannot produce enough feedback to get the observed core size (deBlok 2010; Penarrubia et al. 2012; Vogelsberger et al. 2012).

Another suggestion is that dark matter particles are not collisionless, but weakly self-interacting. Burkert(2000) showed that a core could be produced if the dark matter cross-section per unit mass is about  $\sigma/m \sim 1 \text{ cm}^2 \text{ g}^{-1}$ . The resulting dark matter profile is known as the Burkert profile. Moreover, recent studies in a wide range of galaxies (including dwarf galaxies) report an interesting relation if the dark matter density profile is fitted with a cored density profile:  $\rho_c r_c = \text{constant}$ , where  $\rho_c$  and  $r_c$  are the central density and core radius of the dark matter density profile respectively. This relation is first noticed by Kormendy and

Freeman(2004). They obtained  $\rho_c r_c \sim 100 M_\odot \text{ pc}^{-2}$ , which is almost a constant by using the data from 55 rotation curves in spiral galaxies (Kormendy and Freeman 2004). Later, Spano et al.(2008) analysed the mass distribution of 36 spiral galaxies and got  $\rho_c \propto r_c^{-1.04}$ . Furthermore, Donato et al.(2009) use the data from 1000 spiral galaxies and obtained  $\rho_c r_c \approx 141 M_\odot \text{ pc}^{-2}$  for a wide range of different galaxies. Gentile et al.(2009) also show that this interesting relation can be applied in baryonic component of galaxies. Recently, Salucci et al.(2012) use the kinematic surveys of the dwarf spheroidal satellites of the Milky Way to tighten the relation  $\rho_c \propto r_c^{-a}$  with  $0.9 < a < 1.1$ . However, all the above results are only based on the observations from galaxies. Some studies including the data from galaxy clusters suggest that the product  $\rho_c r_c$  is not really a constant, but depends weakly on the total mass of dark matter halo  $M_{\text{halo}}$ . Boyarsky et al.(2009) and Del Popolo et al.(2013) obtained  $\rho_c r_c \propto M_{\text{halo}}^{0.21}$  and  $\rho_c r_c \propto M_{\text{halo}}^{0.16}$  respectively by extending the sample data to cluster scale. Moreover, recent observation from cluster Abell 611 reports a very large  $\rho_c r_c \sim (2350 \pm 200) M_\odot \text{ pc}^{-2}$  (Hartwick 2012). Although the data from clusters are still not enough to draw any conclusions, these results begin to challenge the universality of dark column density ( $\rho_c r_c = \text{constant}$ ).

In fact, the potential relation between  $\rho_c$  and  $r_c$  suggests that some strong constraints or intrinsic properties may exist in dark matter. Chan(2013a) suggests that the existence of a universal ‘optical depth’  $\tau = \rho_c(\sigma/m)r_c = \text{constant}$  in galaxies can explain the observed relation. However, there is no strong fundamental reason or physical principle why there exists a universal ‘optical depth’. Therefore, it would be very useful to understand the properties of dark matter if we could examine whether the universality of dark matter column density is also true in galaxy clusters.

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However, the mass density profile probed by gravitational lensing cannot provide accurate core radius and central density of dark matter profile. Although the method of weak and strong lensing can give a good direct estimation of projected mass, the 3-D mass function still depends strongly on the dark matter functional form, which is usually assumed to be the NFW profile or generalized NFW profile (Bartelmann and Steinmetz 1996; Mahdavi et al. 2013; Giocoli et al. 2013). However, as mentioned above, the NFW profile deviates from the observed profile significantly for small radius and an additional free parameter is needed to indicate the inner slope of the density profile (Giocoli et al. 2013). Moreover, the generalized NFW profile can give us the density scale only, but not the central density for our analysis. Although Bartelmann and Steinmetz (1996); Shan et al. (2010) are able to obtain a large sample of enclosed cluster mass by strong lensing, we still need to assume some cored-profile (with  $\rho_c$  and  $r_c$ ) to de-project the enclosed mass for our purpose. The result, however, would be highly dependent on the assumed profile.

Alternatively, observations in cluster hot gas provide a good tool to probe the dark matter density profile. Although we need to assume that the hot gas particles are in hydrostatic equilibrium and the distribution is spherically symmetric, we need not assume any cored-profile in the analysis to get the  $\rho_c$  and  $r_c$ . Since the dark matter mass dominates the total mass, the resulting profile can be regarded as the dark matter mass profile. However, the spherical asymmetry, cooling flow in hot gas and the AGN feedbacks may significantly affect the estimated profile. In particular, the cooling flow and AGN feedbacks mainly affect the central part of the density profile in clusters by a factor of 2-4 (Arabadjis et al. 2003; Martizzi et al. 2012). Although these effects are not negligible, it is the only way to obtain a large sample of clusters with corresponding  $\rho_c$  and  $r_c$ . Moreover, we can divide the analysis into subsets such that the cooling-flow clusters can be ruled out in the empirical fits. In this article, we still use this traditional method to get a universal dark matter density profile for 106 galaxy clusters from observations based on the ROSAT All-Sky Survey (Reiprich and Böhringer 2002). By relating the central densities and the core radii of these density profiles, we can test the universality of dark column density for the whole sample and its subsets of clusters.

## 2 MASS PROFILE IN CLUSTERS

The hot gas density profile can be modelled by the King's profile (single- $\beta$  model) (King 1972)

$$n(r) = n_0 \left( 1 + \frac{r^2}{r_0^2} \right)^{-3\beta/2}, \quad (1)$$

where  $n_0$ ,  $r_0$  and  $\beta$  are the fitting parameters. Assuming the hot gas is in hydrostatic equilibrium and spherically symmetric, we get

$$\frac{kT}{m_g} \frac{dn(r)}{dr} = -\frac{GM(r)n(r)}{r^2}, \quad (2)$$

where  $m_g$  is the average mass of a hot gas particle,  $T$  is the hot gas temperature and  $M(r)$  is the enclosed mass pro-

file in a cluster. By combining the Eq. (1) and (2), we get (Reiprich and Böhringer 2002)

$$M(r) = \frac{3\beta kTr^3}{Gm_g(r^2 + r_0^2)}. \quad (3)$$

Here we have assumed that the temperature of hot gas is constant. Although the isothermal profile is not a good assumption for many clusters (Vikhlinin et al. 2006), the estimation of cluster total mass by using Eq. (3) is still a good approximation for our interested region:  $r \leq r_0$  (Allen et al. 2001; Reiprich and Böhringer 2002). It can be justified by assuming an approximated form of hot gas temperature profile (Pointecouteau et al. 2005):

$$T(r) = T_0 + T_1 \left[ \frac{(r/r'_c)^\eta}{1 + (r/r'_c)^\eta} \right], \quad (4)$$

where  $T_0$ ,  $T_1$ ,  $r'_c$  and  $\eta$  are fitting parameters. If we include the temperature variation, the mass and density profile would be respectively modified to

$$M(r) = \frac{kTr}{Gm_g} \left( \frac{3\beta x^2}{1 + x^2} - a \right) \quad (5)$$

and

$$\rho(r) = \frac{kT}{4\pi Gm_g r^2} \left[ \frac{3\beta x^2(3 + x^2)(1 + a)}{(1 + x^2)^2} - a \left( 1 + \eta - \frac{2\eta y^\eta}{1 + y^\eta} \right) \right], \quad (6)$$

where  $a = d \ln T / d \ln r$ ,  $x = r/r_0$  and  $y = r/r'_c$ . The term  $a$  is small for some nearly isothermal clusters. However, for some cool-cored clusters, the temperature variation near the centre is not negligible. Although this value is not universal for all clusters, Hudson et al. (2010) analysed a large sample of clusters and obtained the average values of  $a$  for cool-cored clusters ( $a = 0.24$ ) and non-cool-cored clusters ( $a = 0.08$ ) near the centres of clusters. By using Eq. 6, the term  $a$  would contribute 3% and 25% errors in the estimation of  $\rho_c$  respectively for non-cool-cored clusters and cool-cored clusters. These errors are generally small compared with the observational errors of the required parameters used in the calculation. For the outer region, although we do not have a full set of fitting parameters for 106 clusters, we can extrapolate the result obtained from Allen et al. (2001), which got a narrow range of fitting parameters for several clusters:  $T_0 = (0.40 \pm 0.02)T$ ,  $T_1 = (0.61 \pm 0.07)T$ ,  $r'_c = (0.087 \pm 0.011)r_{2500}$  and  $\eta = 1.9 \pm 0.4$ . The virial radius  $r_{2500}$  is related with another virial radius  $r_{500}$  by  $r_{2500} \approx 0.45r_{500}$  for normal clusters. By using the analysis of  $r_0$  and virial mass of clusters  $M_{500}$  from Chen et al. (2007), we can obtain a relation  $r_0 \propto r_{2500}^{3.54}$ . Therefore, we get  $y \approx 2.5$  when  $r = r_0$  for normal clusters. By putting this estimation to Eq. (6), the percentage error of  $r_c$  is just 6 %, which is much smaller than the observational errors of  $T$ ,  $\beta$  and  $r_0$  (the definition of  $r_c$  will be discussed below). The non-isothermal factor would be large if we consider the total mass at large radius. Therefore, for simplicity, we use Eq. (3) to model all the cluster total mass.

Moreover, some hot gas profiles in clusters cannot be well fitted by the single- $\beta$  model. Chen et al. (2007) re-analysed the data from Reiprich and Böhringer (2002) and discovered that the empirical fits for 49 cluster hot gas profiles can be significantly improved by using a double- $\beta$  model:

$$n(r) = \sqrt{n_{01}^2 \left(1 + \frac{r^2}{r_{01}^2}\right)^{-3\beta_1} + n_{02}^2 \left(1 + \frac{r^2}{r_{02}^2}\right)^{-3\beta_2}}, \quad (7)$$

where  $n_{01}$ ,  $n_{02}$ ,  $r_{01}$ ,  $r_{02}$ ,  $\beta_1$  and  $\beta_2$  are the fitting parameters. By using Eq. (2), the total mass profile becomes

$$M(r) = \frac{3kTr^3}{Gm_g} \frac{n_{01}^2\beta_1\alpha_1^{-3\beta_1-1}/r_{01}^2 + n_{02}^2\beta_2\alpha_2^{-3\beta_2-1}/r_{02}^2}{n_{01}^2\alpha_1^{-3\beta_1} + n_{02}^2\alpha_2^{-3\beta_2}}, \quad (8)$$

where  $\alpha_1 = 1 + (r/r_{01})^2$  and  $\alpha_2 = 1 + (r/r_{02})^2$ .

By using Eqs. (3) and (8), the central mass density of the dark matter for single- $\beta$  model and double- $\beta$  model can be respectively given by

$$\rho_c = \frac{9\beta kT}{4\pi Gm_g r_0^2}, \quad (9)$$

and

$$\rho_c = \frac{9kT}{4\pi Gm_g} \left[ \frac{n_{01}^2\beta_1/r_{01}^2 + n_{02}^2\beta_2/r_{02}^2}{n_{01}^2 + n_{02}^2} \right]. \quad (10)$$

The dark matter core radius  $r_c$  can be regarded as the scale-length of dark matter. It can be defined at which the local dark matter volume density reaches a quarter of its central value (Burkert 2000; Gentile et al. 2009; DelPopolo et al. 2013). We can obtain all the mass density profiles  $\rho(r)$  for clusters from Eqs. (3) and (8) and follow the usual definition to set  $\rho(r_c) = \rho_c/4$  to obtain all values of  $r_c$ .

### 3 DATA ANALYSIS

A large sample of clusters have been examined by ROSAT All-Sky Survey, including the observed parameters  $\beta$ ,  $T$  and  $r_0$  (Reiprich and Böhringer 2002). They obtained some useful relations such as the correlation between the luminosity and total mass of clusters ( $L - M$  relation). Later, Ota and Mitsuda (2004) and Chen et al. (2007) use an improved sample to obtain some other relations, such as  $\beta - r_c$ ,  $T - r_c$ ,  $M - T$  and  $L - M$  relations. However, they didn't analyse the  $\rho_c - r_c$  relation. In Fig. 1, we simply take the improved sample from Chen et al. (2007) and plot  $\log \rho_c$  vs  $\log r_c$ . A tight relation between  $\rho_c$  and  $r_c$  is obtained. By fitting all cluster hot gas with the single- $\beta$  model, the BCES bisector analysis obtains

$$\log \left( \frac{\rho_c}{M_\odot \text{pc}^{-3}} \right) = (-1.47 \pm 0.04) \log \left( \frac{r_c}{1 \text{ kpc}} \right) + (0.75 \pm 0.08). \quad (11)$$

Since a better fits can be obtained by the double- $\beta$  model for 49 clusters (Chen et al. 2007), the same analysis with this improved sample gives

$$\log \left( \frac{\rho_c}{M_\odot \text{pc}^{-3}} \right) = (-1.46 \pm 0.16) \log \left( \frac{r_c}{1 \text{ kpc}} \right) + (0.88 \pm 0.33). \quad (12)$$

A larger uncertainty is resulted in the improved sample because the percentage error in  $r_c$  is generally larger in the double- $\beta$  model, though the  $\chi^2$  value is smaller (Chen et al. 2007). Moreover, in Fig. 1, we compare the scaling relation in galaxies  $\rho_c \propto r_c^{-1}$  obtained by Salucci et al. (2012). Obviously, the data from galaxies and clusters scatter in different parameter space. Therefore, they should correspond to different scaling relations. This suggests that the product  $\rho_c r_c$  is not a universal constant for different scales of structure.

Since cooling flow may affect the mass profile in clusters, we also obtain the  $\rho_c - r_c$  relation for cooling flow clusters and non-cooling flow clusters separately in Fig. 2. The BCES bisector analysis gives

$$\log \left( \frac{\rho_c}{M_\odot \text{pc}^{-3}} \right) = (-1.30 \pm 0.07) \log \left( \frac{r_c}{1 \text{ kpc}} \right) + (0.60 \pm 0.11) \quad (13)$$

for cooling flow clusters and

$$\log \left( \frac{\rho_c}{M_\odot \text{pc}^{-3}} \right) = (-1.50 \pm 0.24) \log \left( \frac{r_c}{1 \text{ kpc}} \right) + (0.96 \pm 0.54) \quad (14)$$

for non-cooling flow clusters. Obviously, the fitted slope for non-cooling flow clusters is very closed to the whole sample.

We also perform a cross-check with another sample from Shan et al. (2010), which studied 27 clusters by combining the X-ray and lensing results. The BCES bisector analysis obtains

$$\log \left( \frac{\rho_c}{M_\odot \text{pc}^{-3}} \right) = (-1.64 \pm 0.10) \log \left( \frac{r_c}{1 \text{ kpc}} \right) + (1.58 \pm 0.21). \quad (15)$$

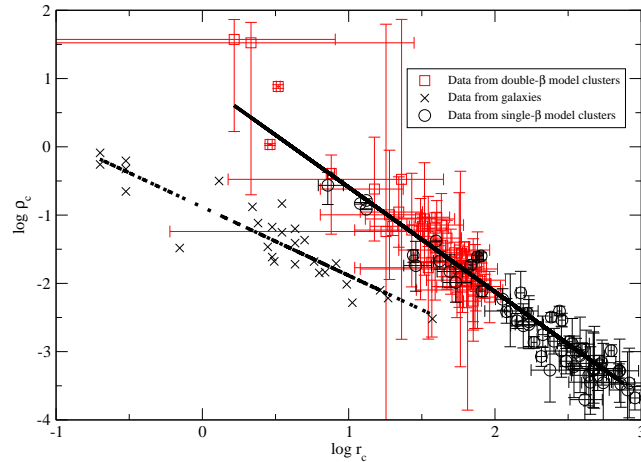
This result generally agrees with our analysis based on 106 clusters (see Fig. 3). The larger slope obtained from Shan et al. (2010)'s sample may be due to the assumption of the single- $\beta$  model used, which has a generally larger  $r_c$ . Moreover, the average hot gas temperature is larger in the sample from Shan et al. (2010) (average  $T = 8.3$  keV) than the sample from Chen et al. (2007) (average  $T = 4.8$  keV). This may also affect the slope in the empirical fits.

Besides, the mean value of  $\rho_c r_c$  for clusters is  $\sim 2000 M_\odot \text{pc}^{-2}$ , which generally agrees with the result from Hartwick (2012), which obtained  $\rho_c r_c \sim 2350 \pm 200 M_\odot \text{pc}^{-2}$ .

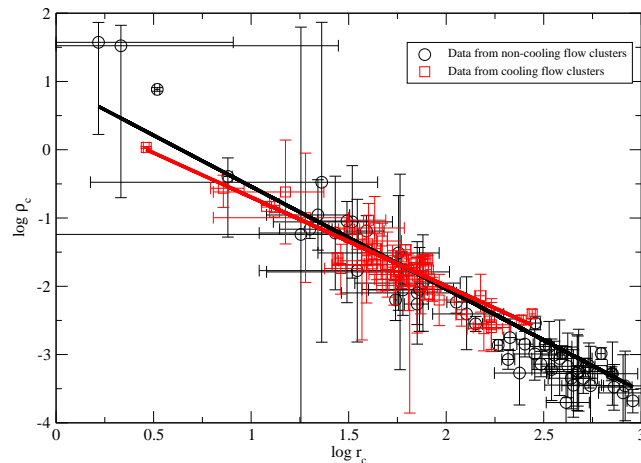
### 4 DISCUSSION AND CONCLUSION

In this article, we obtain the central density and core radius of dark matter in a cluster by using the hot gas profile. The resulting scaling relation is  $\rho_c \propto r_c^{-1.46 \pm 0.16}$ . This result is basically different from that obtained in galaxies:  $\rho_c \propto r_c^{-1}$ . Also, this result gives a tighter scaling relation in clusters compared with the fits from previous studies such as  $\beta \propto r_c^{0.11+0.03}_{-0.02}$  and  $T \propto r_c^{0.03+0.05}_{-0.07}$  (Ota and Mitsuda 2004). On the other hand, the fitted slope for cooling flow clusters (slope =  $-1.30 \pm 0.07$ ) are slightly different from non-cooling flow clusters (slope =  $-1.50 \pm 0.24$ ). This suggests that the cooling flow in clusters may affect the inner structure of dark matter, which is consistent with some suggestions about the baryonic feedbacks in galaxies (Weinberg and Katz 2002; Macciò et al. 2012).

The scaling relation in clusters consists of three basic parameters in hot gas profile,  $\beta$ ,  $r_0$  and  $T$ . These parameters should be independent of each other in hot gas. However, the gravitational interaction between dark matter and hot gas particles relates the three parameters to form a tight scaling relation. Therefore, this scaling relation may reflect some intrinsic properties of dark matter in clusters. If the core formation is due to the self-interaction of dark matter, as suggested by Spergel and Steinhardt (2000), the scattering cross section  $\sigma$  should be related to  $(\rho_c r_c)^{-1}$ . In galactic scale, since  $\rho_c r_c$  is a constant,  $\sigma$  is also a constant for galaxies. This supports the constant (velocity independent) self-interaction



**Figure 1.**  $\log \rho_c$  versus  $\log r_c$  from cluster data (Chen et al. 2007). The units for  $\rho_c$  and  $r_c$  are in  $M_\odot \text{ pc}^{-3}$  and kpc respectively. The fitted slope and the y-intercept for cluster data are  $-1.46 \pm 0.16$  and  $0.88 \pm 0.33$  respectively.



**Figure 2.**  $\log \rho_c$  versus  $\log r_c$  from cooling flow cluster data (squares) and non-cooling cluster data (circles). The units of  $\rho_c$  and  $r_c$  are in  $M_\odot \text{ pc}^{-3}$  and kpc respectively.

cross section scenario (Peter et al. 2013; Rocha et al. 2013; Zavala et al. 2013). However, recent studies in clusters have already revealed that  $\sigma$  should be velocity-dependent ( $\sigma$  decreases as the velocity of dark matter particle increases) (Loeb and Weiner 2011; Chan 2013b). By using our result from cluster data, since  $\rho_c r_c^{1.46}$  is a constant, we have  $\sigma \sim (\rho_c r_c)^{-1} \propto r_c^{0.46}$ . Generally, the velocity of dark matter particle  $v$  increases with  $r_c$  (Rocha et al. 2013). That means  $\sigma$  should increase with  $v$ , which contradicts to the observations and the prediction from velocity-dependent cross section scenario (Colin et al. 2002; Vogelsberger et al. 2012). Therefore, the formation of cores in clusters and galaxies may not be caused by the self-interaction of dark matter particles.

Finally, the two very different scaling relations in galaxies and clusters suggest that some different constraints in dark matter may exist in galaxies and clusters. Therefore, probably there will be no universal dark matter density pro-

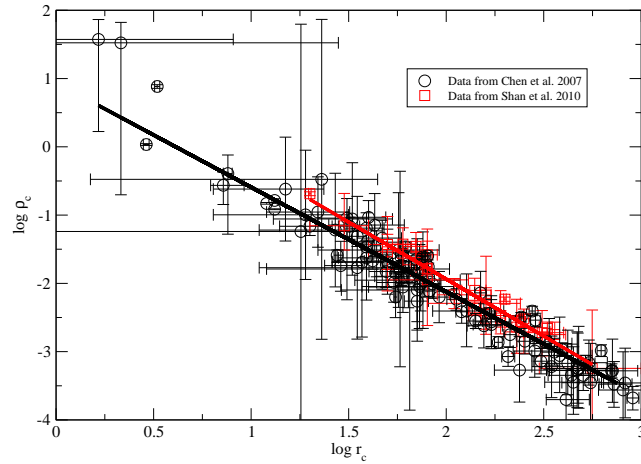
file exist in different scales as predicted by numerical simulations.

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**Figure 3.**  $\log \rho_c$  versus  $\log r_c$  from Chen et al. 2007 data (circles) and Shan et al. 2010 data (squares). The units of  $\rho_c$  and  $r_c$  are in  $M_\odot \text{ pc}^{-3}$  and kpc respectively.

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